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AN INVESTIGATION OF THE EFFICIENCY EFFECTS OF GAS TURBINE FLEXIBILITY WITH BLEED-OFF UTILIZATION

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An Investigation of the Efficiency Effects of Gas Turbine Flexibility with Bleed-Off Utilization.

A Thesis

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Symbols Used

C_v = Specific heat at constant volume, BTU/lb/deg F.

C_p = Specific heat of constant pressure, BTU/lb./deg F.

g = 32.2 ft. per sec. per sec.

J = 778.26 ft. lb per Btu

L = Mechanical work

P = Absolute pressure, lb. per sq. ft.

Q = Heat added or removed, Btu

R = Gas Constant, 53.3 for air

T = Static Temperature, deg. Rankine

T_o = Total temperature, deg. Rankine

v = velocity, ft. per sec.

V = Specific volume, cu.ft. per lb.

W = Flow, lb. per sec.

w = Fuel, lbs per lb. air

INTRODUCTION

Engineering study of the possibilities of the gas turbines as a drive for ship propulsion has been conducted in this country by various large industrial firms. Various arrangements have been considered including the electrical and hydro-mechanical transmissions. The principle difficulty in the development of an efficient gas turbine has been the need for high temprature alloys. This difficulty has been overcome with the introduction of various steel alloys such as 16-256-6 and the Cobalt-Chromium-Columbium alloys. The introduction of these metals has now put gas turbine development on a par with the larger steam plant developments. For ship drives the combustion gas turbine cycle offers efficiency equal to the best maritime steam power plants, which have a maximum over-all efficiencies of about 25%. Weight and space requirements of equipments are a serious factor in this application. The gas cycle eliminates the steam generator, the steam condenser. Without doubt the gas cycle plant will offer a decided decrease in weight and spac e requirements, despite the air compressor, the regenerator, and the gas turbine.

Although gas turbine cycle efficiency is comparable with steam plant efficiency the gas turbine is essentially a point design. Great Savings in weight and space requirements are effected by gas turbine installation. The point design of a gas turbine is that point at which maximum efficiency is obtain ed. Any excessive variance from this designed point

(i.e., partial load) brings a corresponding decrease in the system efficiency. This factor is readily understood when we com e to consider that compressor design in its present stage of development can vary little from its designed load without seriously decreasing the c ompressor efficiency. This also applies to some extent to the turbine efficiency. Various methods have been proposed to increase the efficiency of the gas turbine cycle at partial loa d. The use of separate turbines improves the partial load performance since the compressor and its drive turbine can operate independently at their optimum designed speed regardless of the speed or load requirements of the major power turbine.

*It has been proposed that for ship manoevering conditions that rapid acceleration may be obtained by employing a by-pass valve ahead of the power turbine in operating the compressor at full load. Any excess gas not required by the power turbine for any given load under which it may be operated can be dis charged to the atmosphere through the by-pass valve. The closing of this by-pass valve and thus sending this bleed-off air again to the power turbine will produce maximum torque characteristics. It has been noted that marine propeller torque speed characteristic s are such that during reversing operations while the propeller is

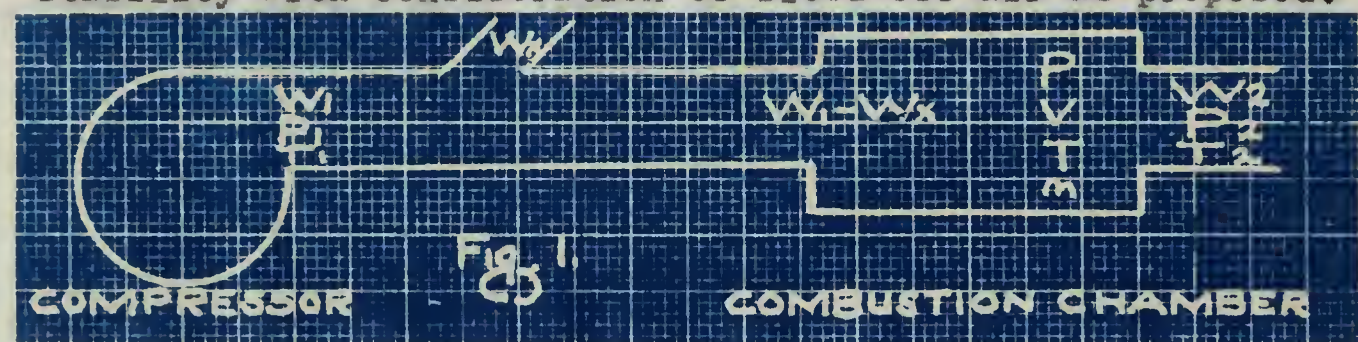
*Dr. J.T. Rettaliata "Gas Turbine" Power January 1944

still rotating ahead the maximum torq ue requirements for reversing occur at approxima tely 40% of the normal shaft speed. The torque speed chara cteristics of the gas turbine can fullfill this condition;the gas turbine will thus have approximately the same acceleration and power characteristics as the present day marine steam turbine.

Present day gas turbine plants yeild good efficiency and flexibility from 75% full load to full load. It is the purpose of this paper to investigate the possibilities of taking this bleed-off air as proposed and passing it through a separate turbine wheel. This turbine wheel will of necessity be isolated from the high temperature wheels due to the high temperature differential. Since compressor stability and compressor-burner stability are important factors in flexibility control the interpreta tion of this ana lysis will yeild the important information which will make bleed-off analysis feasible; this will be undertaken and presented. The added increase in efficiency in the vicinity of ½ load will be investigated with various types of gas turbines as exist in present day development. The author will endeavor to find the most appropriate gas turbine unit which will yeild the optimum conditions for flexibility and efficiency for good partial load characteristics.

I. GENERAL COMPRESSOR STABILITY CONDITIONS WITH BLEED-OFF AIR

A. In the analysis of the gas turbine flexibility as proposed we must of necessity reflect and develop the effects upon the system as a whole, and in measure upon the individual parts of this system. Under given conditions encountered in actual practise it is known that a continuous flow c ompressor coupled with a burner and ful fuel system may become unstable. It is of utmost importance that such c onditions be understood, for often it becomes a major limitation in gas turbine design. The author wishes to present the development of compressor stability with consideration of bleed-off air as proposed.



B. During a transient disturbance we know that the rate of flow entering equals the rate of flow leaving plus the rate of storage. The instantaneous conditions durin g the disturbance are W_1, W_2, W_x and P, wher e:

(1) $W_1 - W_x =$ rate of flow entering

(2) $W_2 =$ rate of flow leaving

(3) $\frac{dW}{dt} =$ rate of storage

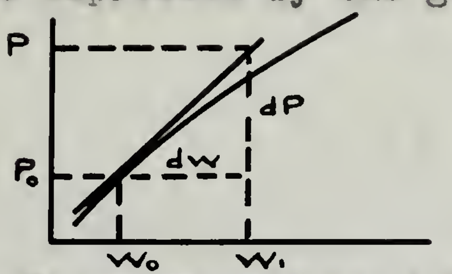
Sinc e $W = \frac{PV}{nRT}$ equation (3) becomes

(3a) $\dfrac{d\left(\frac{PV}{nRT}\right)}{dt}$ = rate of storage

Th us we can sta te B in the following manner:

(4) $W_1 - W_x = W_2 + \dfrac{d\left(\frac{PV}{nRT}\right)}{dt}$

C. The variation of weight flow thru the system with pressure can be expressed by the general curve as shown below.



Now letting W_o, P_o represent any initial steady state conditions, we can express any change in W_1, $W_1 - W_x$ and W_2 with pressure as $\dfrac{dW}{dP}$ where

(1) $\dfrac{dW}{dP} = \dfrac{W_1 - W_o}{P - P_o}$

and,

(2) $(W_1 - W_o) = \dfrac{dW_1}{dP}(P - P_o)$

(3) $(W_1 - W_x - W_o) = \dfrac{d(W_1 - W_x)}{dP}(P - P_o)$

(4) $(W_2 - W_o) = \dfrac{d(W_2)}{dP}(P - P_o)$

We can now state equation B(4) in terms of va riations in flow from the original steady sta te conditions:

(5) $(W_1 - W_x - W_o) = (W_2 - W_o) + \dfrac{d\frac{PV}{nRT}}{dt}$

or,

(6) $\dfrac{d(W_1 - W_x)}{dP}(P - P_o) = \dfrac{dW_2}{dP}(P - P_o) + \dfrac{d\left(\frac{PV}{nRT}\right)}{dt}$

simplifyin g,

(7) $\dfrac{dW_1}{dP}(P - P_o) = \dfrac{dW_2}{dP}(P - P_o) + \dfrac{dW_x}{dP}(P - P_o) + \dfrac{d\left(\frac{PV}{nRT}\right)}{dt}$

At this point let us assume that there is no variation in temperature with pressure. We know this to be untrue, but make the assumption for the sake of simplicity in the analysis. With this assumption, equation C (7) may be simplified to:

$$(8)\quad \frac{dP}{dt} - \frac{nRT}{V}\left(\frac{dW_1}{dP} - \frac{dW_2}{dP} - \frac{dWX}{dP}\right)P = P_o\left(\frac{dW_1}{dP} - \frac{dW_2}{dP} - \frac{dW_x}{dP}\right)$$

Upon examination we see that equation C(8) is in the form of a linear differential equation:

$$(9)\quad \frac{dp}{dt} + AP = B$$

The solution of this type equation can be accomplished by multiplying equation C(9) by E^{AT} to obtain:

$$(10)\quad E^{AT}\frac{dp}{dt} + APE^{AT} = BE^{AT} \quad \text{WHERE}$$

(11)

$$(11)\quad \frac{d(PE^{AT})}{dt} = BE^{AT}$$

$$(12)\quad PE^{AT} = BE^{AT}dt + C_1$$

$$(13)\quad PE^{AT} = \frac{B}{A}E^{AT} + C$$

$$(14)\quad P = \frac{B}{A} + C_1E^{AT}$$

Now applying this intergrating procedure to equation B(8) we obtain:

$$(15)\quad P = P_o + C_1E^{\frac{nRT}{V}\left(\frac{dW_1}{dP} - \frac{dW_x}{dP} - \frac{dW_2}{dP}\right)t}$$

or,

$$(16)\quad P=P_o + C_1E^{\frac{W_oRT}{P_o}\left(\frac{1}{\frac{W_o}{P_o}\frac{dP}{dW_1}} - \frac{1}{\frac{W_o}{P_o}\frac{dP}{dW_x}} - \frac{1}{\frac{W_o}{P_o}\frac{dP}{dW_2}}\right)t}$$

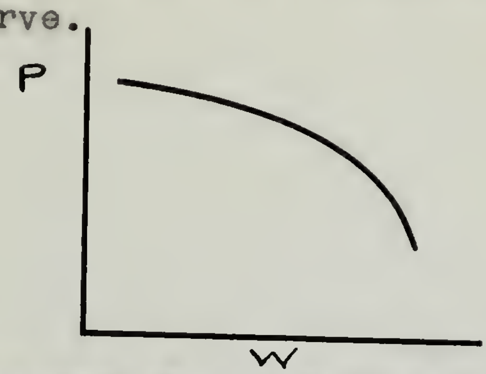
or,

$$(17)\quad P=P_oC_1E^{-\frac{nRT}{v}\frac{dW2}{dp}t}\,E^{-\frac{nRT}{v}\frac{dW_x}{dp}t}\,E^{\frac{nRT}{v}\frac{1t}{\frac{dp}{dW_1}}}$$

Where: $E^{-\frac{nRT}{v}\frac{dW_2}{dP}}$ and $E^{-\frac{nRT}{v}\frac{dW_x}{dp}}$ define the characteristics of the load and $E^{-\frac{nRT}{V}\frac{1t}{\frac{dP}{dW_1}}}$ defines the characteristics of the compressor.

D. Significance of Mathematical Results.

Let us examine a typical Pressure versus Weight of Air Delivered Curve.



The $\frac{nRT}{v} \frac{dW_2}{dP}$ and $\frac{nRT}{v} \frac{dW_x}{dP}$ define the characteristics of the load as stated above, and we know that $\frac{nRT}{v} \frac{dW_2}{dP}$ is always positive irrespective of the load. Therefore in analysis we notice that the $\frac{nRT}{v} \frac{dW_x}{dP}$ will yeild to the system the same effect as that yielded to the system by $\frac{nRT}{v} \frac{dW_2}{dP}$ having negative slope stability for the compressor will result for the exponent of (E) in our equation (17) will be negative and any disturbance to the system will be damped out. Now noting that $\frac{nRT}{v} \frac{dW_x}{dP}$ carries the same sign as $\frac{nRT}{v} \frac{dW_2}{dP}$ it always will tend to stabilize the system, and any disturbance to the system will also tend to damp out. Thus upon the basis of this analysis we are encouraged to investigate further this proposal of bleed-off air for flexability, for it shows decided possibilities.

Let us then determine the significance of the term which defines the characteristics of the compressor, $\frac{nRT}{v} \frac{1t}{\frac{dP}{dW_1}}$. For good stability characteristics this term

must remain negative, for if the term goes thru zero an d the slopebecomes slightly positive, the exponent (E) of equation (17) will become plus and any disturbance will be amplified.

With the promising results of the stability effects of the proposed bleed-ff; it is the purpose of the author to present in this and the following sections the effects of this bleed-off on the efficiencies and outputs of various gas turbine installations as proposed for present day power installations. It is to be remembered that the proposal of bleed-off is one which will be undertaken at loads below full load with an eye to increasing the efficiency by sending this bleed-off air thru a separate turbine wheel on the power unit.

II. GENERAL COMPRESSOR-BURNER STABILITY CONDITIONS WITH BLEED-OFF AIR

A. Referring to diagram in section I; during a transient disturbance the rate of flow entering equals the rate of leaving plus the rate of storage.

Where: W_1, W_2, W_x and P a re the instantaneous conditions during the disturbance and,

(1) $W_1 - W_x =$ rate of flow entering

(2) $W_2 =$ rate of flow leaving

(3) $\frac{dW}{dt} =$ rate of storage

Since $W = \frac{PV}{nRT}$ equation (3) becomes

(3a) $d\left(\frac{PV}{nRT}\right) =$ rate of storage

therefore, from A:

(4) $W_1 - W_x = W_2 + \dfrac{d\left(\frac{PV}{nRT}\right)}{dt}$

The variation of weight flow thru the system with pressure variation can be expressed by the general curve as shown in Section I-C. Letting $W_o, P_o,$ represent any initial steady state conditions, the slope of the curve will be

$$\frac{W_1 - W_o}{P - P_o} = \frac{dW_1}{dP}$$

or, (5) $W_1 = (P - P_o)\frac{dW_1}{dP} + W_o$

(6) $W_2 = (P - P_o)\frac{dW_2}{dP} + W_o$

(7) $W_x = (P - P_o)\frac{dW_x}{dP} + W_o$

and substituting equati ons 5,6, and 7 in 4:

$$(P - P_o)\frac{dW_1}{dP} + W_o = (P - P_o)\frac{dW_2}{dP} + W_o + (P - P_o)\frac{dW_x}{dP} + W_o + \frac{d\left(\frac{PV}{nRT}\right)}{dt}$$

Since both pressure and temperature vary with time the rate of

$$\text{storage} = \frac{d\,\frac{PV}{nRT}}{dt} = \frac{V}{nRT}\frac{dP}{dt} - \frac{PV}{nRT^2}\frac{dT}{dt} \text{ ------------(8)}$$

It is to be noted that the usual pressure-weight flow curves for a compressor are taken for steady state conditions and can not be taken directly to predict transient conditions for they do not take into con sideration inertia effects. One method of including such effects is to put forth the fact that during a transient condition, a compressor will deliver air at a pressure P where

(9) $P = P_1 - I\frac{dW_1}{dt}$ and P_1 is the pressure obtained neglecting inertia effects; $I\frac{dW_1}{dt}$ is the in ertia effect. Now the rate of pressure change in the system becomes:

$$(10)\quad \frac{dP}{dt} = \frac{dP_1}{dW_1}\frac{dW_1}{dt} - I\frac{d^2W_1}{dt^2}$$

Setting up a heat ba lance for the system

$$(11)\quad (W_1 - W_x)\,C_p\,(T - T_1) = wH;$$

where H is the heating value of the fuel. Diff erentiating equation (11) with respect to time; assuming T_1 = constant

$$(12)\quad (W_1 - W_x)C_p\frac{dT}{dt} + C_p(T - T_1)\left(\frac{dW_1}{dt} - \frac{dW_x}{dt}\right) = H\frac{dw}{dt} = H\frac{dw}{dP}\frac{dP}{dt}$$

$$(13)\quad \frac{dT}{dt} = \frac{H}{(W_1 - W_x)C_p}\frac{dw}{dt} - \frac{(T - T_1)}{(W_1 - W_x)}\frac{dW_1}{dt} - \frac{dW_x}{dt}$$

$$dT = \frac{H}{(W_1 - W_x)C_p}\frac{dw}{dP}\,dP - \frac{T - T_1}{(W_1 - W_x)}\frac{(dW_1 - dW_x)}{dP}\,dP$$

Wit h equation (13) substituted in (8) rate of storage =

$$\frac{V}{nRT}\frac{dp}{dt} - \frac{PV}{nRT^2}\left(\left(\frac{H}{(W_1 - W_x)C_p}\frac{dw}{dt} - \frac{T - T_1}{W_1 - W_x}\left(\frac{dW_1}{dt} - \frac{dW_x}{dt}\right)\right)\right)$$

or rate of storage =

$$(14)\quad \frac{1}{n}\left(\frac{V}{RT} - \frac{PV\ H}{RT^2\ (W_1-W_x)\ C_p}\ \frac{dw}{dP}\right)\left(\frac{dP_1}{dW_1}\ \frac{dW_1}{dt} - I\ \frac{d^2W_1}{dt^2}\right) + \frac{PV}{nRT^2}\ \frac{(T-T_1)}{(W_1-W_x)}\left(\frac{dW_1}{dt} - \frac{dW_x}{dt}\right)$$

If t he burn er is operating at velocities high enough to be nea r the acoustic $\frac{W_2(T_o)^{\frac{1}{2}}}{PA}$ = constant

or $W_2 = \frac{P}{K(T)^{\frac{1}{2}}}$ and taking ln we have:

$$\ln W_2 = \ln K + \ln P - \tfrac{1}{2}\ln T$$

taking the differential:

$$(15)\quad \frac{dW_2}{W_2} = \frac{dP}{P} - \frac{1}{2}\frac{dT}{T} \qquad \text{and}$$

$$(16)\quad dW_2 = \frac{W_2 dP}{P} - \frac{W_2}{2}\ \frac{dT}{T}$$

In analysis: At the onset of any transient with the changes of W, T, and P from the steady sta te values are small; thus we can intergrate equation (16_b)

$$(16a)\quad dW_2 = \frac{W_2\ dP}{P} - \frac{W_2}{2T}\left(\frac{H}{C_p\ (W_1-W_x)}\ \frac{dwdP}{dP} - \frac{T-T_1}{W_1-W_x}\ \frac{dW_1-dW_2}{dP}\ dP\right)$$

$$(16b)\quad dW_2 = \left(\frac{W_2}{P} - \frac{W_2}{2T}\ \frac{H}{C_p(W_1-W_x)}\frac{dw}{dP}\right)dP + \frac{W_2\ (T-T_1)\ (dW_1-dW_2)}{2T\ (W_1-W_x)}$$

$$(17)\quad W_2-W_o = \left(\frac{W_o}{P_o} - \frac{H}{2T_oC_p}\ \frac{dw}{dp\ (W_1-W_x)}\right)P-P_o + \frac{(T_a-T_1)}{2T_o}\ \frac{(W_1-W_2)}{(W_1-W_x)}$$

From equ at ion 9

$$(18)\quad P-P_o = (P_1-P_o) - I\ \frac{dW_1}{dt} \quad \text{and for small changes}$$

$$(19)\quad P_1-P_o = (W_1-W_o)\ \frac{dP_1}{dW_1}$$ and substituting equation (19) in equati on (18)

(20) $P-P_o = (W_1-W_o)\,\frac{dP_1}{dW_1} - I\,\frac{dW_1}{dt}$ and from equation (20) and equation (17)

(21) $$W_2-W_o = \left(\frac{W_o}{P_o} - \frac{H}{2T_o}\,\frac{dw}{C_p\,dP\,(W_1-W_x)}\right)\left((W_1-W_o)\frac{dP_1}{dW_1} - I\frac{dW_1}{dt}\right) + \frac{(T_o-T_1)}{2T_o}\,\frac{(W_1-W_2)}{(W_1-W_x)}$$

(21a) $$W_2-W_o = \left(\frac{W_o}{P_o} - \frac{H}{2T_o\,C_p\,(W_1-W_x)}\,\frac{dw}{dP}\right)\left((P_1-P_o) - I\,\frac{dW_1}{dt}\right) + \frac{(T_o-T_1)}{2T_o}\,\frac{(W_1-W2)}{(W_1-W_x)}$$

(22) $$W_2-W_o = \frac{W_o}{P_o} - \frac{H}{2T_oC_p}\,\frac{(W_1-W_o)}{(W_1-W_x)}\,\frac{dw}{dP}\,\frac{dP_1}{dW_1} + \frac{(T_o-T_1)}{2T_o}\,\frac{(W_1-W_2)}{(W_1-W_x)} - I\left(\frac{W_o}{P_o} - \frac{H}{2T_oC_p(W_1-W_x)}\,\frac{dw}{dP}\right)\left(\frac{dW_1}{dt}\right)$$

During a transient condition equation (1) become s

(23) $(W_1-W_o) = (W_2-W_o) + (W_x-W_o) +$ rate of storage

and now substituting equations (22) and (14), we obtain the following result:

(24) $$\frac{d^2(W_1-W_o)}{dt^2} - \frac{d(W_1-W_o)}{dt}\left\|\frac{I\left(\frac{W_o}{P_o} - \frac{H}{2T_oC_p(W_1-W_x)}\frac{dw}{dP}\right) - \frac{P_oV\,(T_o-T_1)}{nRT_o^2(W_1-W_x)} - \frac{1}{n}\,\frac{dP_1}{dW_1}\,\frac{V}{RT_o} - \left(\frac{P_oV\,H}{RT_o^2(W_1-W_x)\,C_p}\right)\frac{dw}{dP}}{\frac{1}{n}\left(\frac{V}{RT_o} - \frac{P_oVH}{RT_o^2(W_1-W_x)}\,\frac{dw}{dP}\right)}\right\|$$

$$+ (W_1-W_o)\left(\frac{1-\left(\frac{W_o}{P_o} - \frac{H}{2T_oC_p}\,\frac{dw}{dP}\right)\frac{dP_1}{dW_1} + \frac{T_o-T_1}{2T_o}}{\frac{1}{n}\left(\frac{V}{RT_o} - \frac{P_oV}{RT_o^2}\,\frac{H}{(W_1-W_x)C_p}\,\frac{dw}{dP}\right)}\right) = 0$$

We shall review for a moment the solution for second order differential equations. Equation (24) may be written

$$(25)\quad \frac{d^2(W_1-W_0)}{dt^2} + a\frac{d(W_1-W_0)}{dt} + b\,(W_1-W_0) = 0$$

and assuming $(W_1-W_0) = x$ thus (25) becomes

$$(26)\quad \frac{d^2x}{dt^2} + a\frac{dx}{dt} + b(x) = 0$$

Let operator $D = \frac{d}{dt}$ and $D^2 = \frac{d^2}{dt^2}$ therefore:

$$(D^2 + aD + b)\,x = 0$$

Setting $D^2 + aD + b = 0$

$$D = \left\{-\frac{a}{2} + \left\|\left|\frac{a}{2}\right|^2 - b\right\|^{\frac{1}{2}}\right\},\quad D = \left\{-\frac{a}{2} - \left\|\left|\frac{a}{2}\right|^2 - b\right\|^{\frac{1}{2}}\right\}$$

$$\left\{D + \frac{a}{2} - \left\|\left(\frac{a}{2}\right)^2 - b\right\|^{\frac{1}{2}}\right\}\left\{D + \frac{a}{2} + \left\|\left|\frac{a}{2}\right|^2 - b\right\|^{\frac{1}{2}}\right\} x = 0$$

$$\frac{dy}{dt} + \left\{\frac{a}{2} - \left\|\left|\frac{a}{2}\right|^2 - b\right\|^{\frac{1}{2}}\right\} y = 0$$

multiplying by E

$$E\,\frac{dy}{dt} + \left\{\frac{a}{2} - \left\|\left|\frac{a}{2}\right|^2 - b\right\|^{\frac{1}{2}}\right\} Ey = 0$$

$$\frac{d\left(yE^{\left\{\frac{a}{2} - \left\|\left|\frac{a}{2}\right|^2 - b\right\|^{\frac{1}{2}}\right\}t}\right)}{dt} = 0$$

$$yE^{\left\{\frac{a}{2} - \left\|\left|\frac{a}{2}\right|^2 - b\right\|^{\frac{1}{2}}\right\}t} = C_1$$

$$\frac{dx}{dt} + \left\{\frac{a}{2} + \left\|\left|\frac{a}{2}\right|^2 - b\right\|^{\frac{1}{2}}\right\} x = C_1E^{\left\{-\frac{a}{2} + \left\|\left|\frac{a}{2}\right|^2 - b\right\|^{\frac{1}{2}}\right\}t}$$

$$x = C_1 E^{\left\{-\frac{a}{2} + \left(\left[\frac{a}{2}\right]^2 - b\right)^{\frac{1}{2}}\right\} t} + C_2 E^{\left\{-\frac{a}{2} + \left(\left[\frac{a}{2}\right]^2 - b\right)^{\frac{1}{2}}\right\} t}$$

and

$$(26) \quad W_1 - W_o = E^{\frac{a}{2} t} \quad C_1 E^{\left\{\left(\left[\frac{a}{2}\right]^2 - b\right)^{\frac{1}{2}}\right\} t} + C_2 E^{\left\{-\left(\left[\frac{a}{2}\right]^2 - b\right)^{\frac{1}{2}}\right\} t}$$

and with the aid of equation (12) we obtain the following result:

$$(28) \quad a = \frac{P_o n}{W_o I} \left(\frac{\frac{IRT_o W_o^2}{VP_o^2} \left(\left[1 - \frac{P_o}{w} \frac{dw}{dP} \frac{T_o - T_1}{2T_o}\right] - \frac{T_o - T_1}{nT_o} \right) - \frac{W_o}{nP_o} \frac{dP_1}{d(W_1 - W_x)} \left(1 - \frac{P_o \, dw \, (T_o - T_1)}{w \, dP \, T_o}\right)}{\left[1 - \frac{P_o}{w} \frac{dw}{dP} \frac{(T_o - T_1)}{T_o}\right]} \right)$$

$$(29) \quad b = \left(\frac{1 - \frac{W_o}{P_o} \frac{dP_1}{d(W_1 - W_x)} \left[1 - \frac{P_o}{w} \frac{dw}{dP} \frac{T_o - T_1}{2T_o}\right] + \frac{T_o - T_1}{2T_o}}{\frac{VI}{nRT_o} \left[1 - \frac{P_o}{w} \frac{dw}{dP} \frac{T_o - T_1}{T_o}\right]} \right)$$

B. The significance of results obtained from the mathematical development in Section II A may be stated as follows. Equation (26) is the stability equation and will apply to this system as developed if the magnitude of the disturbance is not too great. In analysis when (a) remains a plus number any disturbance to the system will be damped out; if (a) becomes a minus number any disturbance to the system will be amplified. Let us further examine our system and the effect of air inertia, for we shall later see that air ine rtia plays a very important part in the compressor-burner stability. In the case of no burning at the burner $T_o = T_1$ and $n = 1$ and investigating this effect on (a) we find

$$(30) \quad a = \frac{IRT_1 W_o^2}{VP_o^2} - \frac{W_o}{P_o}\frac{dP_1}{d(W_1-W_x)}$$

This equation indicates that the effect of air inertia and our bleed off air is to make possible stable operation without burning, even where our slope of $\frac{dP_1}{d(W_1-W_x)}$ is slightly positive.

In stability to the system commences at the point when

$$(31) \quad \frac{dP_1}{d(W_1-W_x)} > \frac{IRT_1 W_o}{VP_o}$$

It is to be remembered that the inertia effect is not an easy effect to calculate for it is a factor inherent to a given system. However it is important to remember that the inertia effect will be essentially the same in case of no burning as

as with burning, provided the flow and pressure are the same. The condition for pulsation for a given value of I is given by the equation, (where $\frac{dw}{dP} = 0$

$$(32)\ \frac{(W_1-W_x)}{P}\ \frac{dP_1}{d(W_1-W_x)} = \frac{nT_o}{T}\left(\frac{W_1-W_x}{P}\ \frac{dP}{d(W_1-W_x)}\right)-\left(\frac{T_o-T_1}{T_o}\right)$$

In analysis for cases of low air inertia the effect of burning is to foster pulsation (assume temperature ratio in combustion chamber equal to 5) in a system that would otherwise be stable with no burning. To further examine this effect of burner temperature on stability let us consider the limit of stability to a system with no air inertia, or

$$\frac{(W_1-W_x)}{P}\ \frac{dP}{d(W_1-W_x)} = 0$$

This yeilds,

$$(33)\ \frac{(W_1-W_x)}{P}\ \frac{dP}{d(W_1-W_x)},\ \frac{T_o-T_1}{T_o}$$ knowing gas stored in burner,

is $\frac{PV}{nRT}$.

$$(34)\ \frac{d\ \frac{PV}{nRT}}{dP} = \frac{V}{nRT}\left(1-\frac{P}{T_o}\ \frac{dT_o}{d(W_1-W_x)}\ \frac{d(W_1-W_x)}{dP}\right)$$

Returning to the original heat balance equation (12) with $\frac{dw}{dP} = 0$

$$(35)\ \frac{d\ T_o}{d(W_1-W_x)} = -\frac{T_o-T_1}{W_1-W_x}$$

This yeilds

$$(36)\ \frac{d\ \frac{PV}{nRT}}{dP} = \frac{V}{nRT}\left(\frac{1+\frac{T_o-T_1}{T_o}}{\frac{(W_1-W_x)}{P}\ \frac{dP_1}{d(W_1-W_x)}}\right)$$

In interpretation, when the conditions in the system are such that equation (33) is satisfied from equation (36) we observe that an increase in pressure to the system is accompanied by an increase in flow weight stored in the burner. This is the desired situation, the stable condition. We may conclude from the above analysis that our proposal of bleed-off air and its resultant effect on compressor-burner stability is one which shows promising application - for the effect is one which tends to maintain system stability.

III. RESULT OF MATHEMATICAL COMPUTATIONS TABLES AND GRAPHS

A. The gas turbine of the constant pressure combustion type comprises three components; the compressor, the combustion chamber, and the turbine. Computations were first carried out for a simple cycle, operating at full load with half load bleed-off. Curves were plotted representing efficiency versus pressure ratios of four, eight and twelve to one respectively. -(Curve I A)- A heat regenerator was added to this cycle and a curve of efficiency versus pressure ratio was also plotted for this cycle variation.- (Curve I B) The efficiency increase over these two systems due to the utilization of bleed-off air via the additional turbine wheel was plotted and is represented by curve I C and I D respectively.

B. The simple cycle with and without regenerator was then subjected to the effects of compressor compounding of 50 and 100%. The efficiency of this variation versus pressure ratio is represented by curves II A_1, and II B_1 for 50% cooling; curve IIA_1 without generator, curve II B_1 with regenerator. The effect of the utilization of bleed-off air is represented by curve II C_1 for the system without regenerator, and curve II D_1 for system with regenerator.

C. The same procedure of labeling has been followed

in the case of compressor compounding 100% cooling. The curves are of the set III and are denoted by subscript 2.

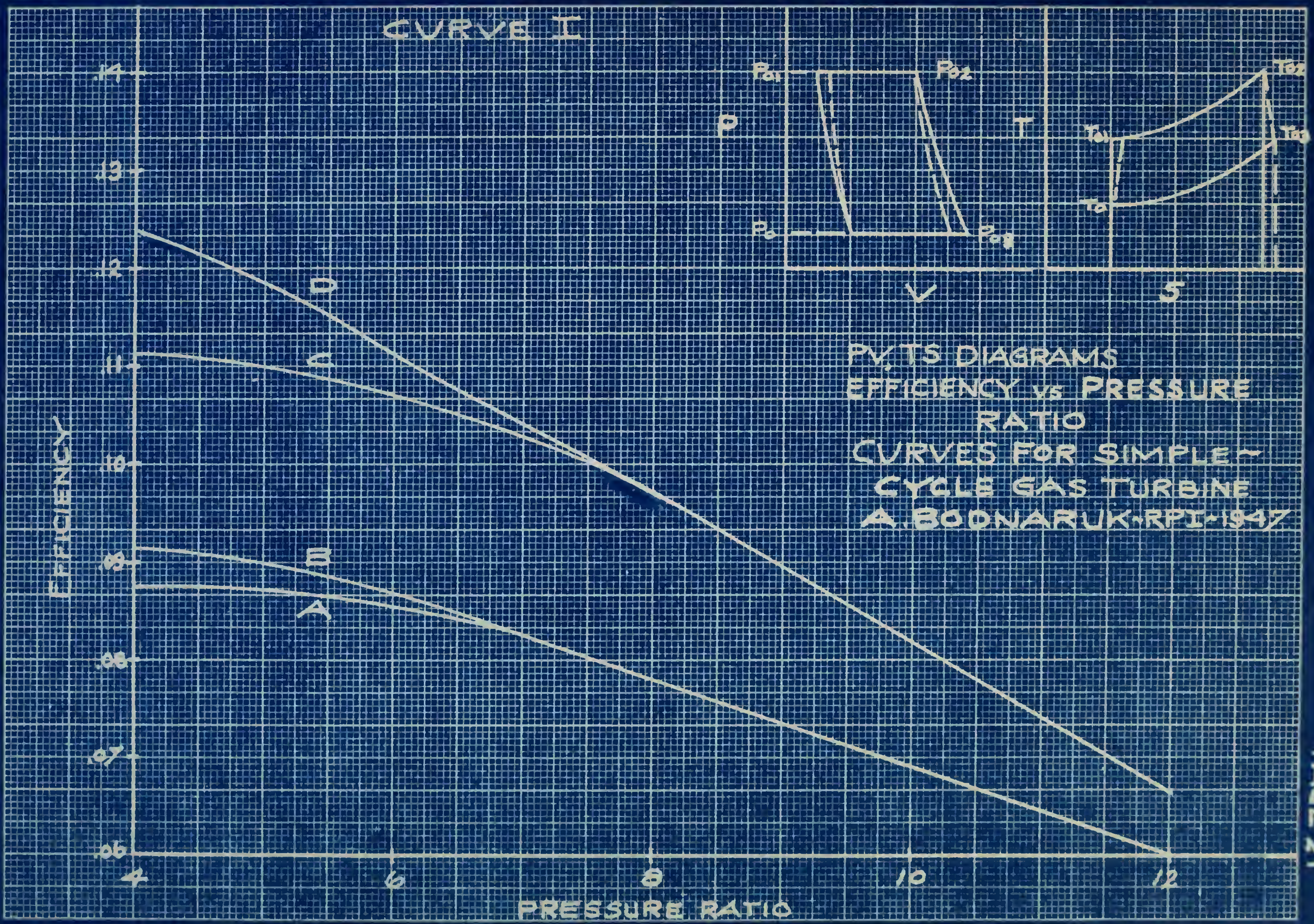
D. We then subject the simple cycle to turbine compounding; i.e. after the first stage expansion the gas is reheated to its original temperature and then expanded in the second stage. The same system of labeling as outlined in paragraph B of this section has been followed. (Curves IV A_3, B_3, C_3, D_3)

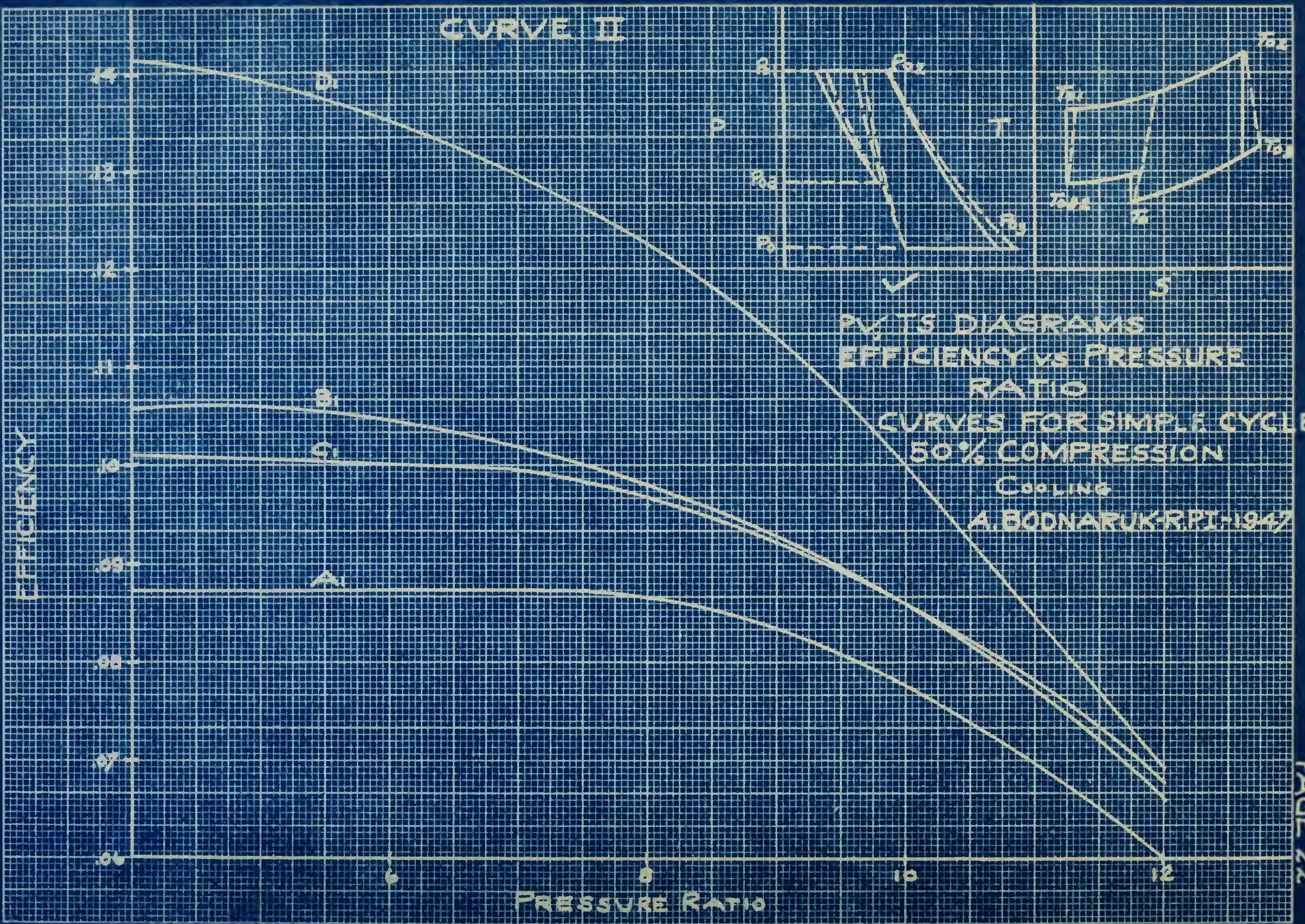
E. Curves V A_4, B_4, C_4, D_4, represent turbine compounding combined with compressor compounding 50% cooling.

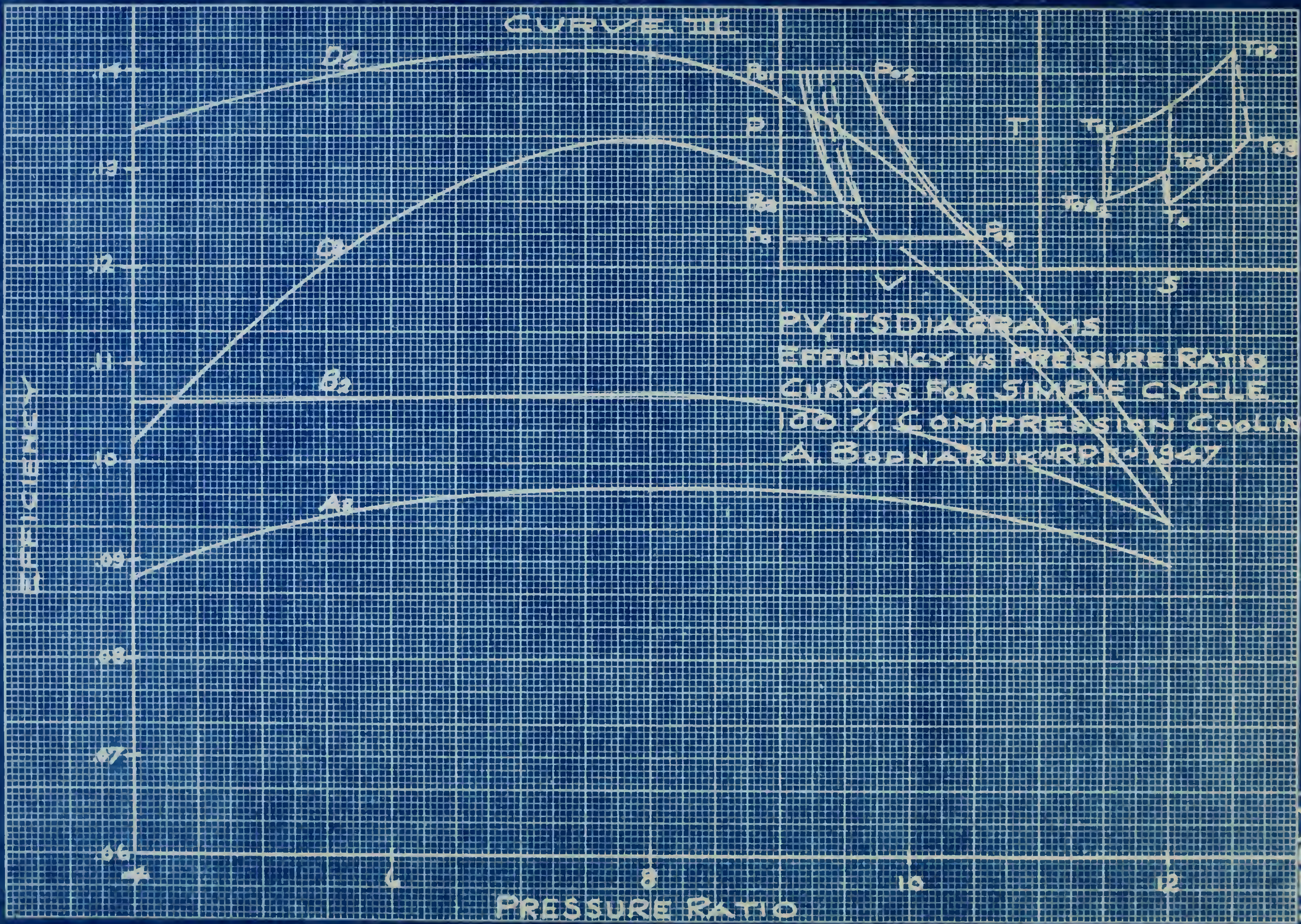
F. Curves VI A_5, B_5, C_5, D_5, represent turbine compounding combined w ith compressor compounding with 100% cooling.

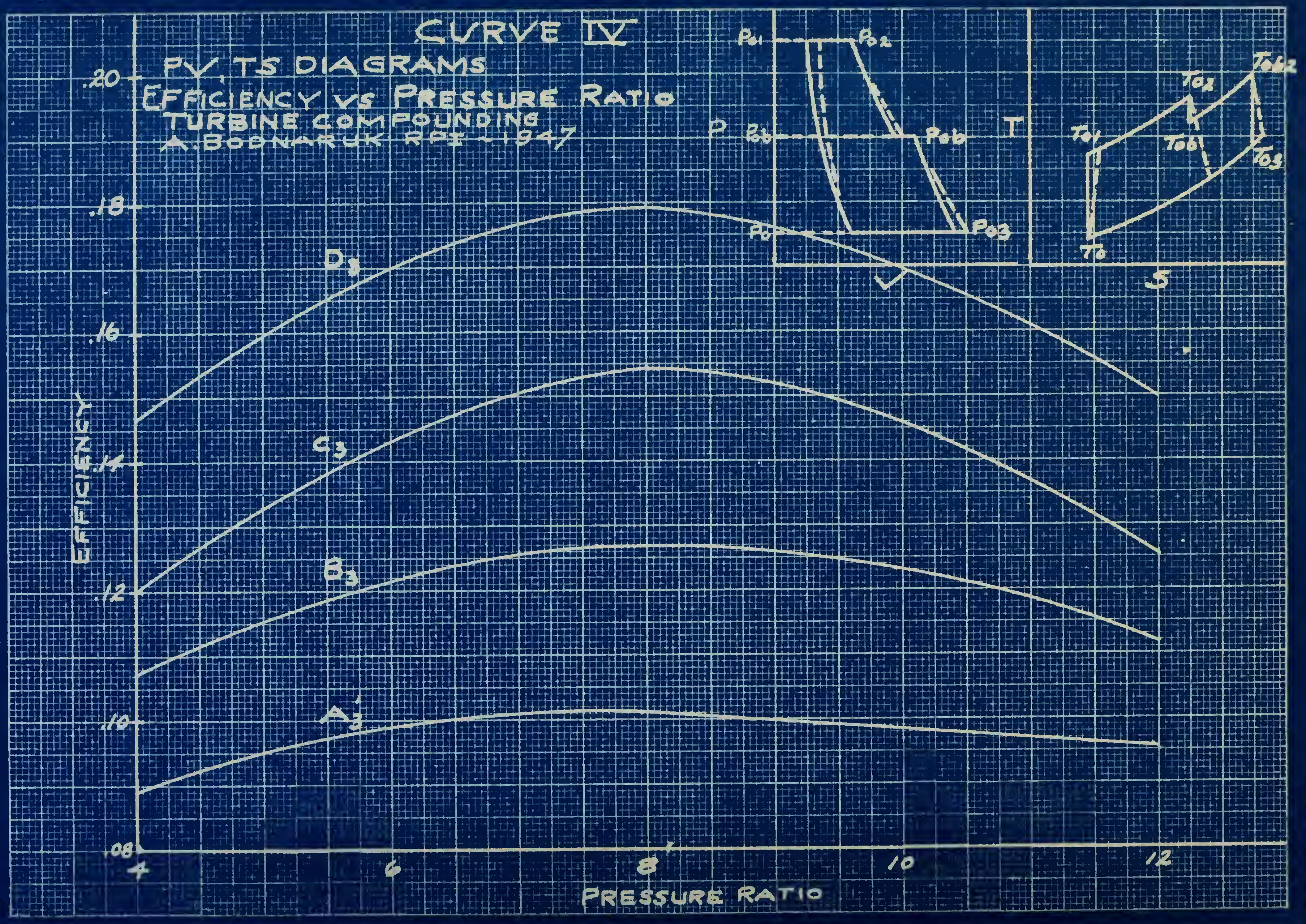
G. It is to be noted that T, S diagrams have been drawn representing the various complex systems on the curves to simplify the understanding of the curves.

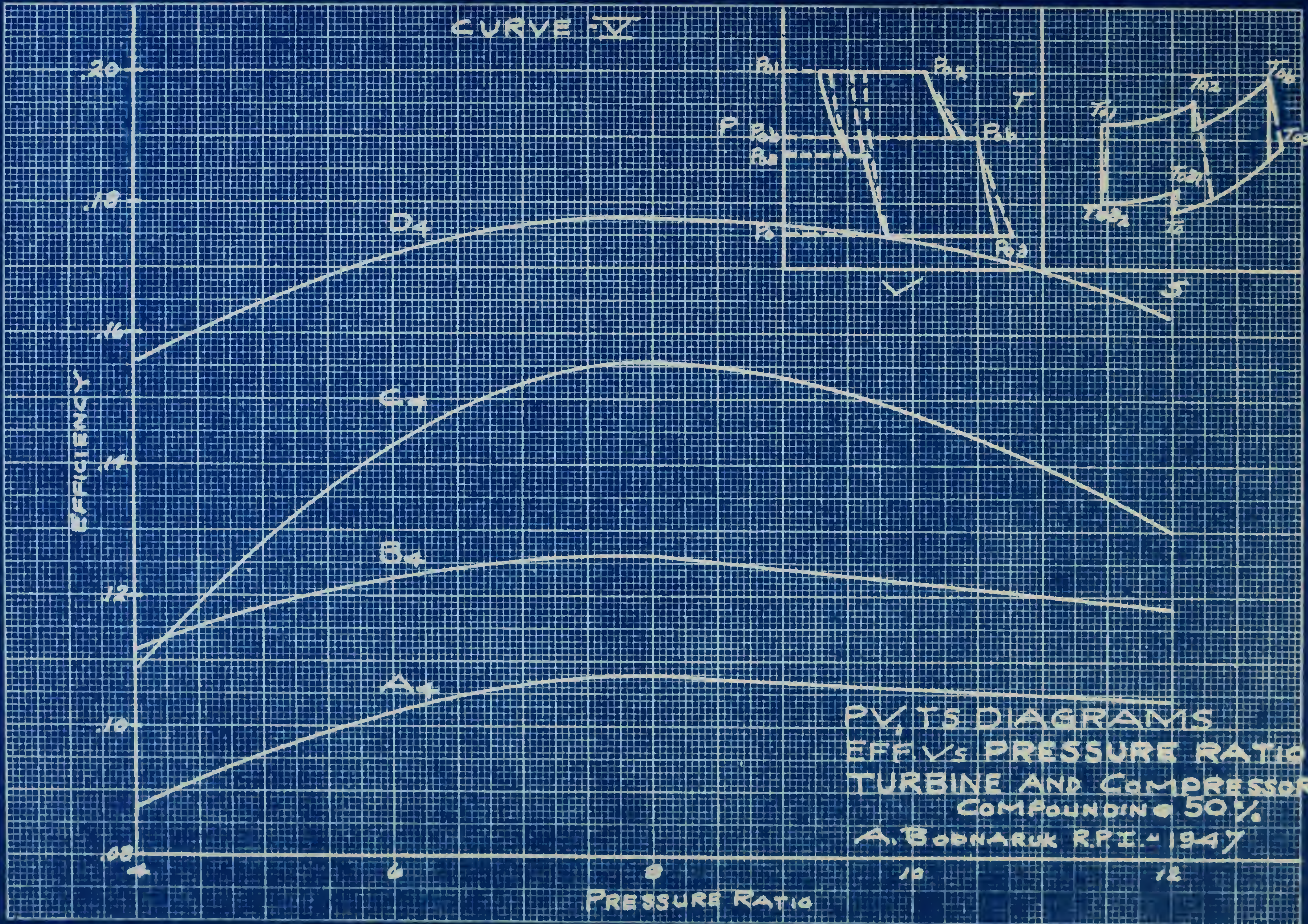
TYPE OF INSTALLATION	SIMPLE			COMPRESSOR COMPOUNDING						TURBINE			COMBINED COMPOUNDING					
				50% cooling			100% Cooling			Compounding			50% cooling			100% Cooling		
PRESSURE RATIO	4	8	12	4	8	12	4	8	12	4	8	12	4	8	12	4	8	12
$T_{o2} = 1500^oR$, $n_t = n_c = .85$ $n_w = .60$																		
Eff. without regenerator (a)	.088	.078	.060	.087	.086	.060	.088	.097	.090	.089	.106	.095	.087	.108	.104	.088	.114	.118
Eff. with regenerator (b)	.091	.078	.060	.106	.097	.066	.106	.107	.094	.111	.127	.118	.112	.126	.118	.111	.129	.129
% increase efficiency	3.40	.000	.000	22.9	13.3	9.40	20.3	9.40	4.60	30.7	20.4	19.0	28.5	16.7	13.2	25.9	13.4	9.20
Eff. with bleed-off use (a)	.117	.097	.067	.101	.116	.068	.102	.133	.094	.120	.155	.125	.109	.156	.130	.110	.157	.160
% increase over (a)	27.7	19.6	11.7	16.1	34.8	13.3	15.9	37.0	3.90	33.0	46.1	31.6	24.0	45.4	24.0	24.8	37.8	38.5
Eff. with bleed-off use (b)	.125	.097	.067	.142	.123	.069	.134	.142	.098	.147	.180	.150	.134	.178	.163	.134	.182	.184
% increase over (a)	42.0	19.6	11.7	62.0	43.0	15.0	52.4	46.5	9.70	65.0	69.9	58.0	54.0	65.0	56.5	52.2	59.6	60.3

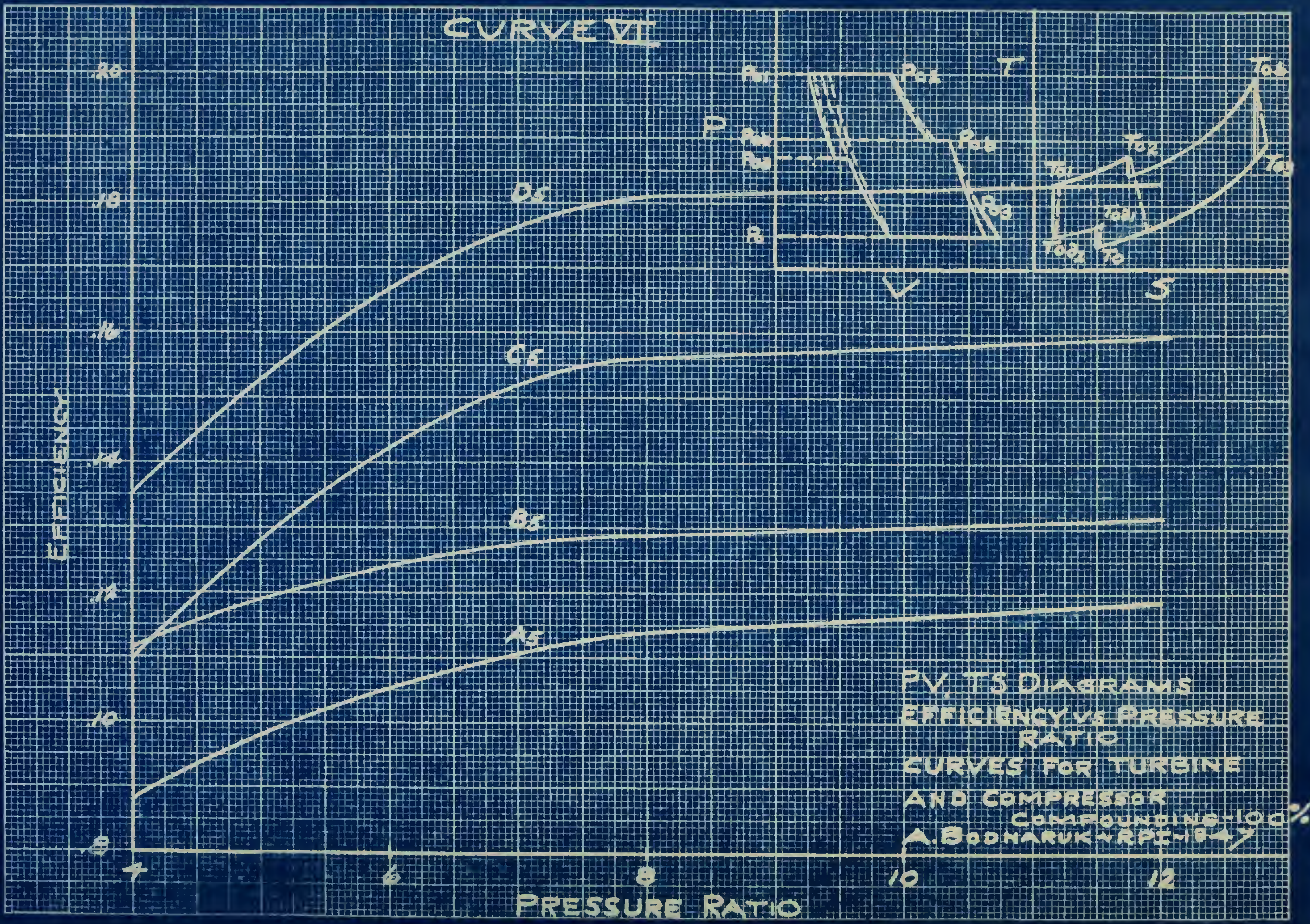

CURVE I
PV, TS DIAGRAMS
EFFICIENCY vs PRESSURE RATIO
CURVES FOR SIMPLE-CYCLE GAS TURBINE
A. BODNARUK-RPI-1947
EFFICIENCY
PRESSURE RATIO
.14
.13
.12
.11
.10
.09
.08
.07
.06
4
6
8
10
12
A
B
C
D
P
V
T
S


CURVE II
D₁
B₁
C₁
A₁
P
T
V
S
PV, TS DIAGRAMS
EFFICIENCY vs PRESSURE
RATIO
CURVES FOR SIMPLE CYCLE
50% COMPRESSION
COOLING
A. BODNARUK-R.P.I.-1947
EFFICIENCY
PRESSURE RATIO
.14
.13
.12
.11
.10
.09
.08
.07
.06
6
8
10
12


CURVE III
PV, TS DIAGRAMS
EFFICIENCY vs PRESSURE RATIO
CURVES FOR SIMPLE CYCLE
100 % COMPRESSION COOLING
A. BODNARUK - RPI - 1947
EFFICIENCY
PRESSURE RATIO
.14
.13
.12
.11
.10
.09
.08
.07
.06
4
6
8
10
12
D2
C2
B2
A2
P
V
T
S


CURVE IV
PV, TS DIAGRAMS
EFFICIENCY VS PRESSURE RATIO
TURBINE COMPOUNDING
A. BODNARUK RPI - 1947
EFFICIENCY
PRESSURE RATIO
.08
.10
.12
.14
.16
.18
.20
4
6
8
10
12
D_3
C_3
B_3
A_3'
P
T
S
P_{01}
P_{02}
P_{0b}
P_{03}
P_0
T_0
T_{01}
T_{02}
T_{0b}
T_{0b2}
T_{03}


CURVE V
EFFICIENCY
.08
.10
.12
.14
.16
.18
.20
4
6
8
10
12
PRESSURE RATIO
D_4
C_4
B_4
A_4
P
V
T
S
PV, TS DIAGRAMS
EFF VS PRESSURE RATIO
TURBINE AND COMPRESSOR
COMPOUNDING @ 50%
A. BODNARUK R.P.I. - 1947


CURVE VI
EFFICIENCY
PRESSURE RATIO
.20
.18
.16
.14
.12
.10
.8
4
6
8
10
12
Ds
Cs
Bs
As
P
V
T
S
PV, TS DIAGRAMS
EFFICIENCY vs PRESSURE RATIO
CURVES FOR TURBINE AND COMPRESSOR
COMPOUNDING-100%
A. BODNARUK-RPI-1947

IV. EFFECT OF BLEED-OFF AIR UTILIZATION ON THE VARIOUS SYSTEMS

A. Effects on the Simple Cycle.

The results of the calculation with turbine and compressor efficiency equal to .85, the turbine efficiency of the seperate wheel equal to .60, and T_{02} equal to 1500 degrees R are given in Table A. It is to be noted that the temperature of the exhaust gases is comparatively high and it is cle ar that utiliza tion of this exhaust heat would increase the efficiency of the basic cycle. This is possible by transferr ing as much heat as possible to the compressed air delivered to the combustion chamb er. Thus by reducing the fuel consumption, the overall efficiency of the cycle is improved, as is evidenc ed by curve I -B at a pressure ratio of 4 to 1. There is no increase in efficiency at pressure ratio 8 or 12 due to the high temperature of the air after compression and the fact that to limit the size of the regenerator we assume a mean temperature differential of 120 degrees. Examining the effect of the utilization of bleed-off air in this simple cycle , we find that the predominant increase occurs in the vicinity of the pressure ratio of 4 to 1; shown by comparison of curve IA to IC, and by comparison of curve IB to ID. As the pressure ratio is further increased

the efficiency effect (i.e., net output B.T.U. per lb. of air) drops off consid erably. At the higher pressure ratios, (i.e. 8 and 12 to 1) the compression work increases rapidly, accounting for the sharp decrease in efficiency. This is an important factor to remember in considering the effect of bleed-off utilization upon the systems in general. The increase in efficiency to the basic systems is a dependent function, and depends upon the efficiency tendancies of the basic system. When this basic efficiency is such as to give large quantities of air at end of compression temperature for bleed-off utilization; the corresponding increase in efficiency will be high, as shown by curve ID as compared to curve IA. As pressure ratio is increased the efficiency of the basic c cycle drops off (curve IA); thus less air is available for bleed-off utilization and the corresponding increase in efficiency drops off. It is to be noted that as the pressure ratio of compression is increased the temperature of the air leaving the compressor becomes higher and due to this higher temperature we are able to obtain more work from a given amount of bleed-off air, but the difficulty encountered lies in the fact that as the pressure ratio is increased in the basic cycle the overall cycle efficiency drops and less and less air is available for bleed-off. Therefore we can see that the amount of air available for bleed-off utilization is the more important consideration. This factor is in evidence in the examination of curves IA and IC.

B. Effects on Compressor Compounding.

Referring to the curves II and III and checking Table A we find that compounding the compressor yields results showing a substantial increase in the efficiency of the respective cycle. This substantial increase in efficiency being greater the more efficient the intercooling (i.e.- curves IIA_1 compared to $IIIA_2$; curves IIB_1 to $IIIB_2$) In analysis the increase is obtained from the reduction of compression work, since the turbine work is essentially the same. In analyzing curve IIA_1, 50% intercooling, it is to be remembered that in intercooling the temperature of the air entering the combustion chamber (T_{01}) is reduced; thus an increase must be made in the quantity of fuel supplied to raise the temperature of the compressed air to 1500 degrees R. Therefore for pressure ratios up to 8 to 1 the efficiency of this cycle remains constant, the reduction in compression work being balanced by the increase in fuel quantity. This reduction of compression work overcomes the increase in fuel in the system of 100% intercooling and thus the efficiency increases to the pressure ratio of 8 to 1. Again the increase in efficiency (curves IIB_1 and $IIIB_2$) due to regenerator effect is more effective at the lower pressure ratios where the net output in B.T.U. per lb. of air is greatest. In the case of 50% intercooling the effect of utilization of bleed-off air is greater at lower pressure ratios (curve ID_1) and

and drops off rapidly when the pressure ratio is increased above 4 to 1. Ther net output in B.T.U. per lb. of air in this region of increased pressure ratio drops accordingly. The stabilization effect to the system due to the regenerator is noted in curve $IIIB_2$. The increase in efficiency in the cycle with 100% intercooling shows marked gains at the pressure ratio of 8 to 1, dropping off rather sharply after this peak is reached (curve $IIIC_2$) This general trend is followed in the system with 100% intercooling and regeneration (curve $IIID_2$) but is more or less modified in effect due to the complexity of the system; for the cycle is not then dependent on one function for increase in efficiency. It is seen therefore that the effects of c compounding regeneration, and utilization of bleed-off air are a great benefit to the system, for a considerable increase of out-put is gained.

C. Effects on Turbine Compounding.

In this section the expansion in the turbine has been accomplished in two stages. The exhaust gases from the first stage are passed to a second bombustion chamber where fuel is again introduced and burned at constant pressure, thus restoring the original maximum temperature to the air. Upon investigati on this results in an increase of overall efficiency (curve IVA_3). The added increase in efficiency due

to regenerator effect is quite uniform (curve IV-B$_3$) and follows the same general trend of curve IV-A$_3$. These curves show
show a marked improvement in efficiency; the values at pressure ratio of 4, the efficiency is increased from .089 to .111, an improvement of 30.7 per cent. It is to be noted that when curve IA is compared to curve IV-A$_3$ at a pressure ratio of 8 the increase in efficiency due to turbine compounding is from .078 to .106, an increase of 35.8 percent. In this system the added increase d ue to utilization of bleed-off air (curve IV-C$_3$) follows the same general efficiency trend of the simple turbine compounding system (curve IV-A$_3$). The accumulative effect of adding the increase in regenerator effect (curve IV-D$_3$) brings the efficiency to a high level, following the general trend of curves IV-B$_3$ and IV-C$_3$. It is interesting to note that the peak effic iency of this system with its various modifications occurs in the vicinity of pressure ratio of 8 to 1. The dotted curve above curve IV-D$_3$ is the curve IV-D$_3$ modified by recalculation, found necessary due to inaccuracy of average C_p in this region.

D. Effects on Turbine Compounding and Compressor Compounding.

In consideration of the benefits obtained from separate compounding of the compression (series curves II and III) and turbine compounding (curve IV), it is proposed in this section of the paper to consider a cycle in which the compressor air is compounded and the turbine gases are compounded and the

turbine gases are compounded. To make reasonable comparisons in this study of f lexibility the same assumptions were made with respect to turbine efficiency, compressor efficiency, temperature of gases, etc. In this study curve V represents 50% compressor intercooling and turbine compounding and Curve VI represent 100% compres or intercooling and turbine compounding. Since this cycle proposal is by far the most interesting the individual flexibility effects will be considered.

1. In the consideration of the effect of intercooling, curve V-A_4 represents 50% compressor intercooling and turbine compounding and curve VI-A_5 represents 100% compressor intercooling and turbine compounding. It is noted that as the pressure ratio of the systems is increased in the vicinity of 8 to 1 there is a marked improvement in efficiency. It is also noted that this increase in the higher pressure ratios is not very sensitive to the efficiency of intercooling. The relative increase in output is noteworthy. We may conclude that with combined com pounding we can expect a profound improvement in efficiency. Output increase is considerable even when we consider the intercooling of 50%.

2. As has been discussed in Section IV A the regenerator effect is important in the consideration of any gas turbine unit. Curve V-B_4 represents the regenerator effect upon turbine compounding and curve VI-B_5 represents the effect upon the system of compressor compounding 100% intercooling. As

suggested in section IV-A we are assuming a mean temperature differential of 150 degrees to limit the size of the regenerator. In analysis the overall efficiencies of the cycles are improved but the effec t on output is not very great. This method of heat utilization merely reduces the fuel consumption by the transfer of heat from the exhaust gases to the air prior to entering the combustion chamber.

3. In the study of the effect of bleed-off utilization, curve V-C_4 represents the bleed-off effect upon the 50% intercooling compressor and turbine compounding, curve V-B_4 represents the same system as curve V-C_4 with the effect of a regenerator added; compressor and turbine compounding, curve VI-D_5 represents the same system as curve VI-C_5 with regenerator effect added. In analysis as the pressure ratio is increased in the systems the efficiency of the base cycle goes up, thus making larger quantiti es of air at higher temperatures available for bleed-off utilization. As the pressure ratio of 8 to 1 is reached the optimum efficiency and output comditions are reached and the effect then levels out, very much in keeping w ith the tendencies of the base cycle. We may thus conclude that with combined compounding in the vicinity of pressure ratios of 8 to 1 we may expect a marked increase in efficiency and output.

E. Practical Aspects of Results and Recommendations.

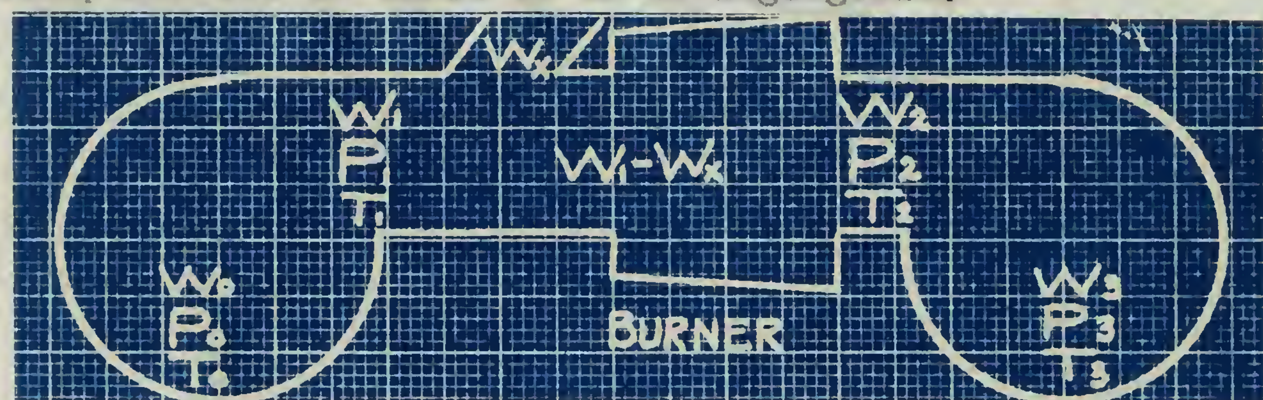
In review of the results obtained from the theoretical analysis of s tability conditions of the gas turbine unit, the mathematical data, and the interperatation of these results; it appears that bleed-off utilization would serve a very practical purpose. The flexibility of the unit would merely be a matter of preparing a schedule of bleed-off and readjusting existing schedules to conform with the new requirements. The problem does not resolve itself so simply, for although good results were obtained f rom bleed-off utilization with regard to efficiency and output increase; these gains were predominant in units of varying degree of complexity with additions of compressor intercoolers and turbine compounding combustion chambers. One of the greatest advantages of the gas turbine in its present day develepement has been the factor simplicity, the unit so assembled with all the apparatus on one shaft and no water required in any of its various stages. The factor of complexity could easily be condoned if the unit would justify the additional expense via added increase in output and efficiency. This is not in the realm of practicality in bleed-off utilization for it is noted that in the units sh showing good gains in efficiency and output the pressure ratio is in the vicinity of 8 to 1. The assumption made relative to compressor efficiency equal to .85 would not

would not hold at such high pressure ratios. Thus the results obtained as shown in curves V and VI would not hold in view of present day compressor development. The gains in efficiency and output would thus have to be modified and calculations with this factor in mind show a drastic drop in efficiency an d output. After careful consideration it is the opinion of the author that in view of present day units and development the proposal of bleed-off utilization, and thus flexibility, would not warrant the expense of the additional turbine wheel.

In keeping with the important thought of gas turbine flexibility the author wishes to propose other avenues in search of this goal. Wet compression in the high pressure ratios merits the investigation of further analysis. Centrifugal turbines have aroused some interest and a complete analysis would be an important contribution. Composite compressors, combining axial flow stages with centrifugal stages, present interesting possibilities for gas turbine units of greater flexibility.

V. SAMPLE CALCULATIONS

It is the purpose of this section to illustrate the method and principles used in the calculation of mathematical material in this paper. At this point it would be appropriate to discuss the accuracy of the results obtained. For ease of com putation the specific heat of the exhaust gases is assumed to be that of air; although this method of computation does not give absolute accuracy; the errors involved due to this assumption are so small as to be negligible.



Given data: $T_o = 530^oR$ $n_c = .85$

$T_{o2} = 1500^oR$ $n_t = .85$ $\frac{P_{o1}}{P_o} = 4$

k (for compression) = 1.395 , k (for expansion) = 1.35

cp (" ") = .241 , cp (" ") = .2644

The gen e ral energy equation

$$(1) \quad dL + dQ = C_v dT + \frac{1}{J} PdV + \frac{1}{J} VdP + \frac{1}{Jg} v\, dv$$

In analysis if we use total temperatures the velocity term in the general energy equation will fall out. Since we can assume in this basic development that the compression is adiabatic($dQ = 0$)

$$(2)\quad dL = C_v dT + \frac{1}{J} PdV + \frac{1}{J} VdP$$

Now knowing $Pv = RT$

$$(3)\quad PdV + VdP = RdT \text{ and substituting in (2)}$$

$$(4)\quad dL = C_v dT + RdT = (C_v + R)\, dT = C_p dT$$

VA Simple Cycle

A. Thus the work done by the compressor in compressing the air is

$$(5)\quad dL = W_1 \int_{T_o}^{T_{o1}} C_p dT$$

$$(6)\quad L = W_1 C_p (T_{o1} - T_o) \text{ ideal}$$

$$(7)\quad L_c = \frac{W_1 C_p (T_{o1} - T_o)}{n_c} \text{ actual}$$

$$(8)\quad \frac{T_{o1}}{T_o} = \left(\frac{P_{o1}}{P_o}\right)^{\frac{k-1}{k}} ; \quad T_{o1} = T_o \left(\frac{P_{o1}}{P_o}\right)^{\frac{k-1}{k}}$$

$$T_{o1} = 530\,(4)^{.283} = 787^\circ \text{ R ideally}$$

$$(9)\quad L_c = \frac{.241\,(257)}{.85} = 73 \text{ BTU/lb. air compressed}$$

B. Turbine Calculations

k for air at $1500^\circ R = 1{,}350$, $C_p = .2644$

$$(1)\quad T_{o2} = \left(\frac{P_{o2}}{P_{o3}}\right)^{\frac{k-1}{k}} ; \quad \frac{1500}{T_{o3}} = (4)^{.259} ; \quad T_{o3} = \frac{1500}{1.432} = 1045^\circ R$$

$$(2)\quad L_t = C_p\, n_t\, (T_{o2} - T_{o3}) = (.2644)\,(.85)\,(455)$$

$$L_t = 102.6 \text{ BTU/lb air compressed}$$

Now net work available for po wer and bleed off is

$$(3)\quad 102.6 - 73 = 29.6 \text{ BTU/lb. air}$$

and since we are assuming that we will be at half load there

will be 14.8 BTU/lb. air available for power purposes and 14.8 BTU/lb air available for b leedoff to atmosphere.

C. In order to calculate the efficiency of the cycle we must find the weight of fuel necessa ry to sustain the cycle under conditions of half load. Fuel used will be kerosene, with a L.H.V. equal to 1900 BTU/lb.

The weight of air for net power purposes will be (from work term balance)

$$(1)\quad 73\ W_1 + 29.6\ W_x = 102.6\ (W_1 - W_x)$$

$$W_x = .224 \text{ lb air/sec}$$

Writing a heat balance for combustion we obtain

$$(2)\quad W_1 - W_x \quad {}^{T_{o2}}_{T_{o1}}\ C_p dT = w\ H = 1900w$$

$$(3)\quad C_p \text{ average} = \frac{.241 + .264}{2} = .257$$

$$(4)\quad w = \frac{.776\ (.257)(668)}{19000} = .00727 \text{ lb fuel/lb air/sec}$$

$$n_{cycle} = \frac{C_{p1}\ (1 + w)\ n_t\ (T_{o2} - T_{o3}) - \frac{C_{p2}}{n_c}\ (T_{o1} - T_o)}{C_{p3}\ (1 + w)\ (T_{o2} - T_{o1}) \text{ actual}}$$

$$n_{cycle} = \frac{14.8 \text{ BTU/lb air/sec}}{168.5 \text{ BTU/lb air sec}} = .0885 \text{ a t half load}$$

D. From the 14.8 BTU/lb air bleed off we will try to reclaim some work via turbine wheel as proposed. We will assume efficiency equals .60 for this seperate turbine wheel. We know that the temperature of this bleed-off air is the discharge temperature of the compressor.

$$T_{o1} = 832^\circ \text{ R actual},\ T_o = 530\ ^\circ \text{ R}$$

$\frac{P_{o1}}{P_o} = 4$, $C_p = .241$, $W_{x2} = .112$ lb. air/sec.

Thus with this given data the additional amount of work obtainable is

(1) $L = W_{x2}\, C_p\, n_w\, (T_{o1} - T_o) = (.112)(.6)(.241)(302) = 5.75$ BTU lb air

The cycle efficiency with utilization of bleed-off air as proposed w ill now be

(2) $n_{cycle} = \frac{(14.8 + 5.75) \text{ BTU/lb air}}{168.5 \text{ BTU/lb air}} =$ at half load

E. Percentage increase of effic iency

(1) $\frac{.0245}{.0885} = .277$

F. Percentage inc rease of output

(1) $\frac{5.75}{14.8} = .389$

$\bar{V}$ B Compounding the Compression - 50% Intercooling

A. We shall accomplish the compression in two stages such that $\frac{P_{oa}}{P_o} = \frac{P_{o1}}{P_{oa}}$ or $P_{oa} = (P_o\, P_{o1})^{\frac{1}{2}}$, which yields minimum work. The intercoolin g reduces the temperature of the air at P_{oa}. When the temperature of the air is reduced to the temperature of the air at P_o it has been subjected to what may be called perfect cooling.

(1) $P_{o_a} = 2$; $T_{oa\,ideal} = T_o \left(\frac{P_{oa}}{P_o}\right)^{\frac{k-1}{k}} = 530\ (1.216) = 640^\circ R$

(2) $L_{c1} = \frac{W_1\, C_p\, (T_{oa} - T_o)}{n_c} = \frac{.241\ (110)}{.85} = 31.3$ BTU/lb air

With 50 per cent intercooling $T_o = 595^\circ$ R

(3) $T_{o1} = 595\ (1.232) = 732^{\circ}\ R$

(4) $L_{c2} = \dfrac{.241\ (137)}{.85} = 38.8$ BTU /lb air

(5) $L_{c1} + L_{c2} = 70.1$ BTU/lb air

B. Turb ine Calculations

(1) (1) from VA-B $L_t = 102.6$ BTU/lb air

Net Work

(2) $102.6 - 70.1 = 32.5$ /lb air

C. $T_{o1\ actual} = 756^{\circ}\ R$

(1) $69.2\ W_1 + 35.5\ W_x = 102.6\ (W_1 - W_x)$

$W_x = .257$ lb air/sec/lb compressed air

Solving for fuel air ratio as is VA-C-(2),(3)

(4) $w = \dfrac{.743(.257)(756)}{19000} = .00765$ lb fuel/lb air/sec

(5) $n_{cycle} = \dfrac{16.25 \text{ BTU/lb air /sec}}{186.5 \text{ BTU/lb air/sec}} = .0870$ at half load

D. Solving for increase in turbine work by use of bleedoff air

(1) $L = .178\ (.241)(218.5) = 6.61$ BTU/lb air

(2) $n_{cycle} = \dfrac{(16\ .2 + 6.61)}{186.5} = .101$

E. Percentage increa.e of efficiency

(1) $\dfrac{.014}{.0870} = .161$

F. Percentage increase of output

(1) $\dfrac{6.61}{16.2} = .409$

G. Solving for increase in turbine work due to bleed-off air plus regenerator effect.

(1) $L = 6.61 + 3.62 = 10.23$ BTU/lb air

(2) $n_{cycle} = \frac{26.43}{186.5} = .142$

H. Percentage increase of efficiency

(1) $\frac{.054}{.087} = .62$

I. Percentage increase of output

(1) $\frac{10.23}{16.2} = .63$

V C <u>Compounding the Compression - 100% Intercooling</u>

A. Compressor

(1) $L_{c1} = 33$ BTU/lb air for 1st stage

(2) $L_{c2} = 33$ BTU/lb air for 2n d stage

(3) $L_{c1} + L_{c2} = 66$ BTU/lb a ir

B. Turbine Calculation

(1) from VA-B $L_t = 102.6$ BTU lb air

Net Work

(2) $102.6 - 66 = 36.6$ BTU/lb air

C. $T_{c1\ actual} = 666.5^{\circ}$ R

(1) $66\ W_1 + 36.6\ W_x = 102.6\ (W_1 - W_x)$

$W_x = .262$ lb air/sec/lb air

Solving for fuel air ratio as VA-C (2), (3)

(4) $w = \frac{.738\ (257)\ (833.5)}{19000} = .0083$ lb fuel/lb air/sec

(5) $n_{cycle} = \frac{18.3 \text{ BTU /lb air/sec}}{208 \text{ BTU/lb air/sec}} = .088$

D. Solving for increase in turbine work by using bleed-off air as proposed

(1) $L = \frac{.131(.241)(136.5)}{} = 3.04$ BTU/lb air

(2) $n_{cycle} = \frac{(18.3 + 3.04)}{208} = .102$

E. Percentage increase of efficiency

(1) $\frac{.014}{.088} = .159$

F. Percentage increase in output

(1) $\frac{3.04}{18.3} = .166$

V D Compounding the Turbine

A. The large surplus of air needed to keep the maximum temper ature of the h ot gases to a reasonable level is in analysis a detriment to high efficiency. The fuel supply is limited to an amount needed to raise the temperature of the air from T_{d} t o T_{o2}. We now arrange the expansion in the t urbine in two stages. By passing the exhaust gases from the first stage to a heating chamber the original maximum temperature is restored to the gases. In this manner we can investigate the possibilities of a more efficient way in which to use the air.

(1) $P_{ob} = \frac{P_{o2}}{2}$; this ratio assumed because it yields higher efficiency than the geometric mean between P_2 and P_3

(2) $\frac{T_{o2}}{T_{ob}} = \left(\frac{P_{o2}}{P_{ob}}\right)^{\frac{k-1}{k}}$; $\frac{1500}{T_{ob}} = (2)^{.259}$; $T_{ob} = \frac{1500}{1.197} = 1250^{\circ}$ R

$T_{ob\ actual} = 1294^{\circ}$ R

(3) $L_{t1} = C_p\, n_t\, (T_{o2} - T_{ob}) = .2644 \times .85 \times 250 = 56.3$ BTU/lb air

(4) $L_{t2} = C_p\, n_t\, (T_{o2} - T_{ob}) = 56.3$ BTU/ lb air

(5) $L_{t1} + L_{t2} = 112.6$ BTU/lb air

Now net work available for power and bleed-off is

(6) $112.6 - 73 = 39.6$ BTU/lb air

therefore 19.8 BTU/lb air is available for bleed-off to atmosphere.

B. Weight of air for net power purposes will be:

(1) $73\,W_1 + 39.6\,W_x = 112.6\,(W_1 - W_x)$

$W_x = .262$ lb air/sec

C. Weight of fuel used

(1) $w = \dfrac{.738\,(.257)\,(910)}{19000} = .00905$ lb fuel/.738 lb air/sec

(2) $n_{cycle} = \dfrac{19.8}{222} = .089$ at half load

D. Efficiency calculation with regenerator

(1) $n_{cycle} = \dfrac{24.6}{222} = .111$ at half load

V E Regenerator with Compressor and Turbine Compounding

A. In this section of the paper we will give consideration to the utilization of the heat valve in the exhaust gases in heating the discharge air from the compressor before it enters the burner chamber. In thus heating the compressed air to a temperature near that of the exhaust we have to supply a decreased amount of fuel to bring it to the final maximum temperature. To limit the size of the heat inter-changer we will limit the temperature exhaust gases to 120 degrees. To illustrate the problem method let us compute the added efficiency due to regenerator effec t for combined compressor and turbine compounding. (VD- B3)

(1) $T_{o1} = 666.5^\circ$ R

(2) $T_{o3} = 1294^\circ$ R

(3) $T_{ob} = 1144^\circ$ R; temperature of air after regenerator effect.

Th e amount of fuel necessary to raise .708 lb air from 1144° R to 1500° R equa ls

(4) $w = \dfrac{.708\ (.257)\ (.356)}{19000} = .0032$ lb fuel/.708 lb air

which adds hea t equivelent of 6.2 BTU/lb air to the cycle

(5) .0032 (19,000) = 6.2 BTU/lb air

T hus

(6) $n_{cycle} = \dfrac{(23.3 + 6.2)}{253} = .115$

or percentage increase in efficiency of 26 per cent. Additional turbine wheel effect for bleed off air

(7) $n_{cycle} = \frac{34.05}{25.3} = .134$

or percentage increase over (a) in effic iency of 52.2 per cent

B. Net work available for power and bleed-off in the various cycles.

(1) Simple cycle: 112.6-73 = 39.6 BTU/lb air

(2) Compressor interc ooling, 50 percent: 112.6-70.1 = 42.5 BTU/lb air

(3) Compressor intercooling 100 percen t: 112.6-66 = 46.6 BTU/lb air

C. Fuel weight and efficiency calculations

(1) Simple cycle

(a) $73\ W_1 + 39.6\ W_x = 112.6\ (W_1 - W_x)$

$W_x = .260$ lb air/sec

(b) $w = \frac{(.74)\ (.257)(668)}{19000} = .0068$ lb fuel/lb air/sec

(c) $n_{cycle} = \frac{19.2}{216} = .089$ at half load

(d) L(increased) = (.13)(.241) (302) = 6.65 BTU/lb air

(e) $n_{cycle} = \frac{25.85}{216} = 120$

Percentage increase of efficiency = .33

Percentage increase of output = .346

(2) Cycle with compressor intercooling, 50%

(a) $70.1\ W_1 + 42.5\ W_x = 112.6\ (W_1 - W_x)$

$W_x = .279$ lb air/sec

(b) $w = \frac{(.721)(.257)(756)}{19000} = .00738$ lb fuel/lb air/sec

(c) $n_{cycle} = \frac{21.25}{243} = .0875$ at half load

(d) L(increase) = $(.139)(.241)(218.5) = 5.2$ BTU/ lb air

(e) $n_{cycle} = 21.25 + 5.2 = .109$

(f) Percentage increase of efficiency = .24

(g) Percentage increase of output = .234

(3) Cycle with compressor intercooling, 100%

(a) $66\ W_1 + 46.6\ W_x = 112.6\ (W_1 - W_x)$

$$W_x = .292 \text{ lb air/sec}$$

(b) $n_{cycle} = \frac{23.3}{25.3} = .0885$

(c) $w = \frac{(.708)(.257)(833.5)}{19000} = .0080$ lb fu el/lb air/ sec

(d) L(increase) = $(.96)(.241)(136.5) = 4.55$ BTU/lb air

(e) $n_{cycle} = \frac{23.3 + 4.55}{253} = .11$

(f) Percentage increase of efficiency = .13

(g) Percentage increase of output = .195

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